Particles and Rotational motion

Chapter 6

System of particles and Rotat motion.

System :

A system is a collection of very large no. of particles which mutually interact with each other

Internal Forces :

The mutual forces exerted by the particles of a system on one another are called internal forces.

External forces :

The outside force exerted on an object by any external agency is called an external force

Centre of mass :

Centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion

Centre of mass v/s centre of Gravity

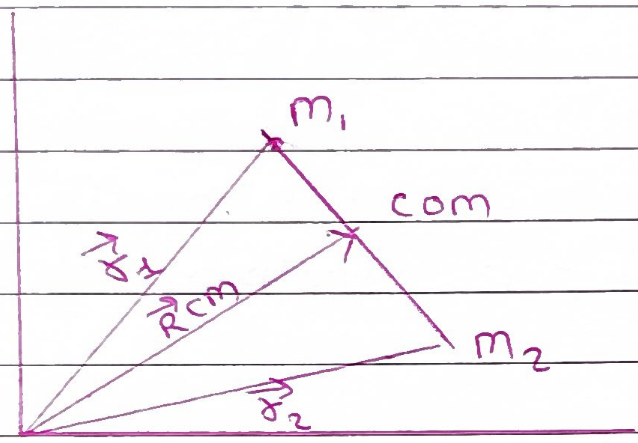
The centre of gravity is a point at which the resultant of the gravitational force on all the particle of the body acts thats is a point where whole weight may be assumed to act.

Centre of mass of a two particle system :-

$$\boxed{\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}}$$

$$\therefore x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$



Velocity of centre of mass is constant in the absence of external force

$$\vec{F}_{tot} = M\vec{a}_{cm}$$

Let $\vec{F}_{tot} = 0$

$$M\vec{a}_{cm} = 0 \qquad \because m \neq 0$$

$$\vec{a}_{cm} = 0$$

$$\therefore \frac{d\vec{v}_{cm}}{dt} = 0$$

$$\vec{v}_{cm} = \text{constant}$$

"In the absence of any external force, the centre of mass of the System moves with uniform velocity"

→ This is Newton's first law of motion

Momentum conservation and centre of mass motion

Let $F_{total} = 0$

$m\vec{a} = 0$

$m\frac{d\vec{v}}{dt} = 0$

$\frac{d}{dt}(m\vec{v}) = 0$

$\frac{d\vec{p}}{dt} = 0$

$$\boxed{\begin{array}{c} P = \text{constant} \\ \vec{P}_{cm} = \text{constant} \end{array}}$$

✫ Turning Effect of force ÷

To open a door, we apply force on its handle. The door turn on its hinges The larger the force, more is its turning effect.

→ This turning effect of force is called moment of force or torque It depend on

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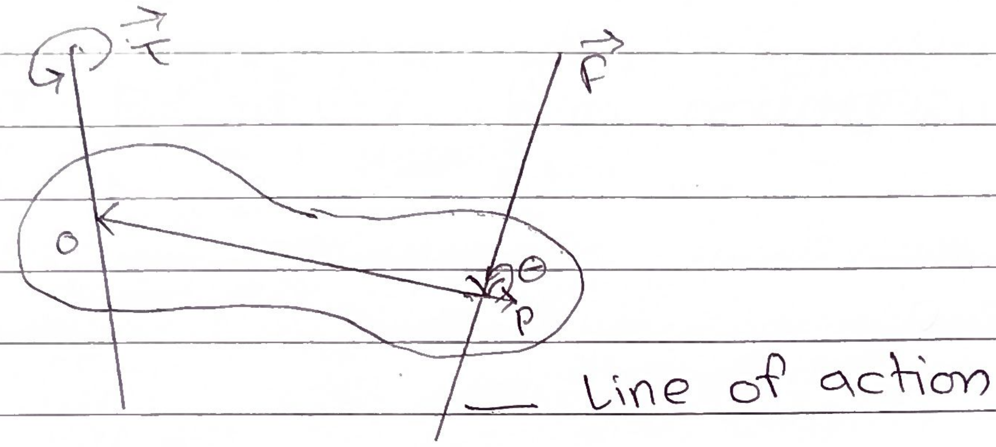
i) $\tau \propto r$
ii) $\tau \propto F$

$\therefore \boxed{\tau = rF} \rightarrow \tau = rF\sin\theta$
$\rightarrow \boxed{\vec{\tau} = \vec{r} \times \vec{F}}$



Sign Convention :-
The moment of force is taken +ve if the turning tendency of the force is anticlockwise and -ve if it is clockwise

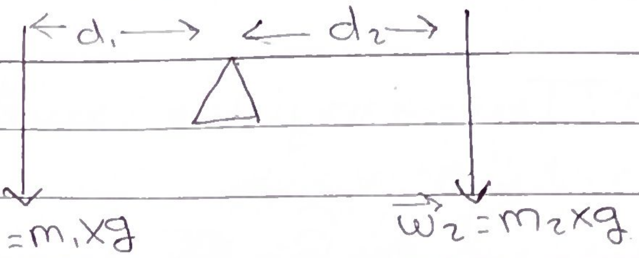
→ Torque is also known as moment of force.

Principle of moments :-
When a body is in rotational equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point or the algebric sum of moments about any point is zero.

Anticlockwise moment
=
Clockwise moment

$F_1 d_1 = F_2 d_2$

$w_1 d_1 = w_2 d_2$

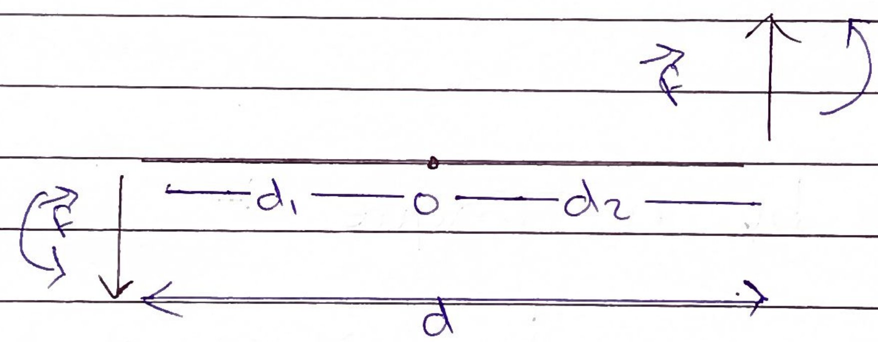


$\boxed{\text{Load} \times \text{Loadarm} = \text{Effort} \times \text{Effort arm}}$

Note : This is also known as lever principle.

Couple :-

A pair of equal and opposite forces acting on a body along two different lines of action from a couple

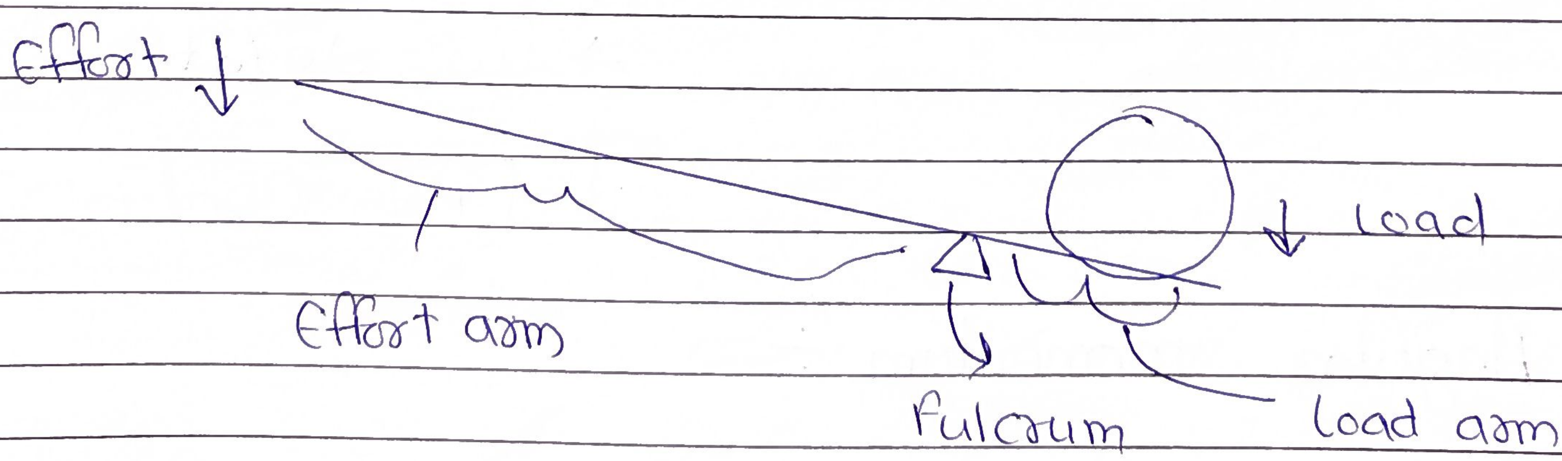


Anticlockwise moment

$= fd_1 + fd_2$

$= f(d_1 + d_2)$

$= fd$



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Work done by a Torque :

In angular motion

$f \approx \tau$

$d \approx \theta$

$$\boxed{W = \tau\theta}$$

τ = constant

In linear motion

$W = fd$

$$\boxed{W = \int_{x_1}^{x_2} f\,dx}$$

If τ is variable

$$W = \int_{\theta_2}^{\theta_1} \tau\, d\theta$$

Power delivered by a Torque :

$$P = \frac{dW}{dt}$$

$$\boxed{P = \frac{d}{dt}(\tau\theta)}$$

If τ = constant

$$\boxed{P = \tau\left(\frac{d\theta}{dt}\right)}$$

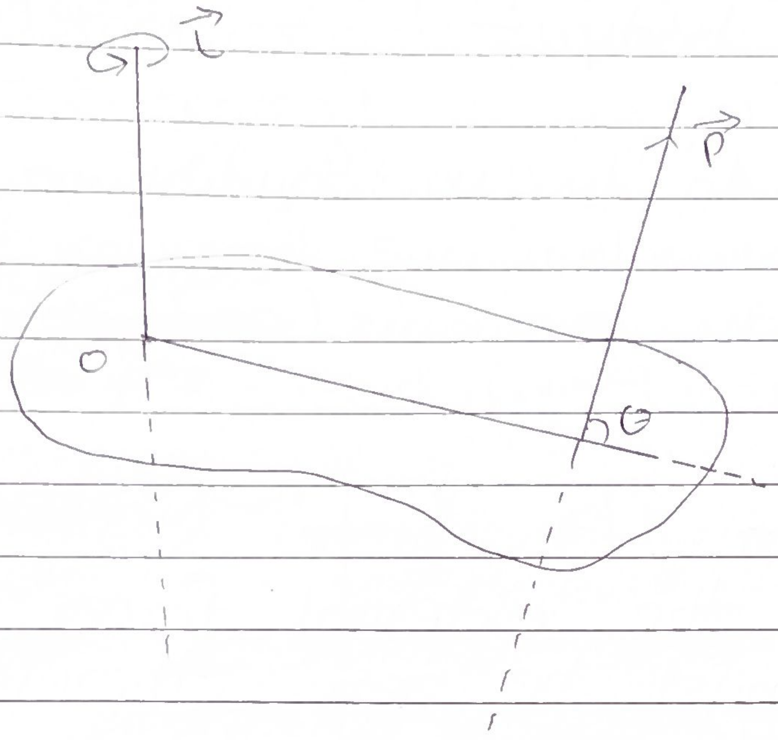
Now → angular velocity

→ $\omega = d\theta/dt$

$$\boxed{P = \tau\omega}$$

Angular momentum :

It is defined as the moment of linear momentum of the particle about that axis



case I = $\theta = 0°$, $L = 0$

case II = $\theta = 90$, $L = rp$(ma

case III = $\theta = 180°$, $L = 0$

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$$\boxed{L = rp\sin\theta}$$

Relation betn L and τ :-

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + (\vec{v} \times \vec{p})$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

$\because$ $\boxed{\vec{v} \times \vec{p} = 0}$

Equilibrium of a rigid body :-

A rigid body is said to be in equilibrium if both the linear momentum and angular momentum of the body remains constant with time.

1) Translational Equilibrium :-

The resultant of all the external forces acting on the body must be zero otherwise they would produce linear acceleration.

$$\vec{F} = m\vec{a}_{cm}$$

If $F = 0$

$$m\vec{a}_{cm} = 0$$

$\because m \neq 0$

$$\vec{a}_{cm} = 0$$

$$\boxed{\vec{v}_{cm} = \text{constant}}$$

Rotational Equilibrium :-

The resultant of torques due to all the forces acting on the body about any point must be zero otherwise they would produce angular acceleration

$$\Sigma \vec{\tau}_{ext} = 0$$

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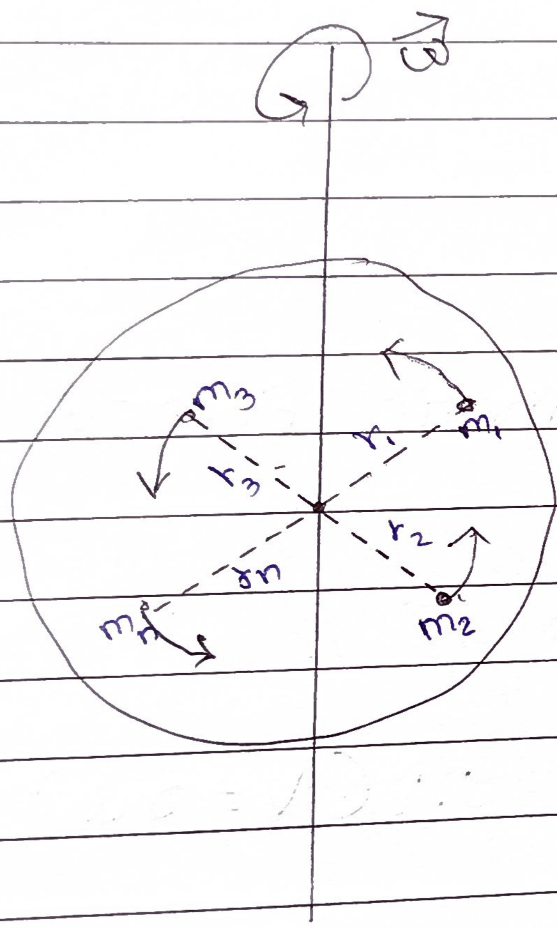
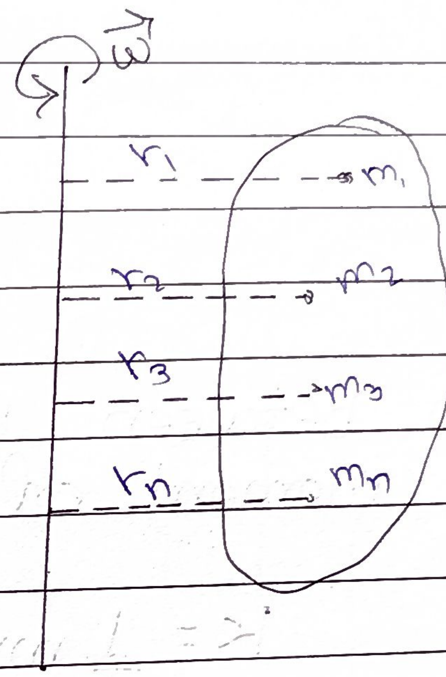
Moment of inertia :-

The moment of inertia of a rigid body about a fixed axis is defined as the sum of the product of the masses of the particles of the body and the squares of their respective distance from the axis of rotation.





$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

d. formula $= [ML^2]$
unit $= kgm^2$

Physical Significance of moment of Inertia :-

→ The mass of a body resist a change in its state of linear motion, it is a measure of its inertia in linear motion.

Factor on which moment of inertia depend :-

1) Mass of the body
2) Size and shape of the body

Relation between Rotational Kinetic Energy and moment of inertia :-

$$K = \frac{1}{2} mv^2$$

$$K = \frac{1}{2} m(r\omega)^2 \qquad \because (V = r\omega)$$

$$K = \frac{1}{2} mr^2 \omega^2$$

$$\boxed{K = \frac{1}{2} I\omega^2} \qquad (\because I = mr^2)$$

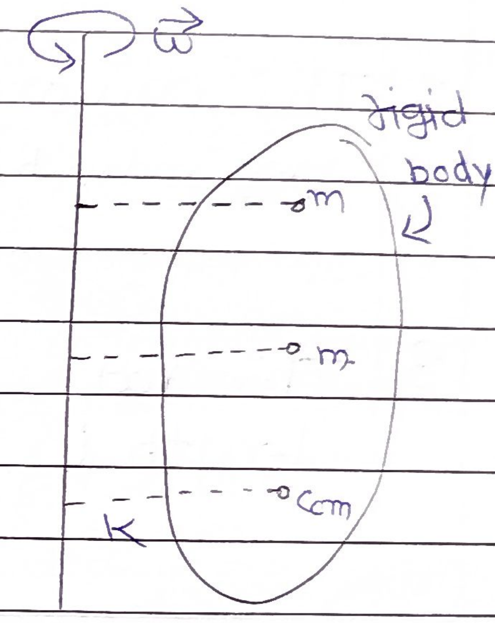
Radius of Gyration :-
For any body capable of rotation about a given axis, it is possible to find a radial distance from the axis where if whole mass of the body is concentrated, its moment of inertia will remain unchange. This radial distanse is called radius of gyration, It is denoted by 'K'

$I = mr_1^2 + mr_2^2 + \dots + mr_n^2$

$I = n\frac{(mr_1^2 + mr_2^2 + \dots + mr_n^2)}{n}$

n = no. of particles of the body/ ~~sty~~ system of Particles

$I = (nm)\frac{(r_1^2 + r_2^2 + \dots + r_n^2)}{n}$



Total moss of the object

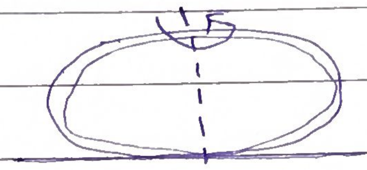
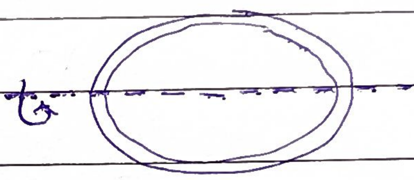
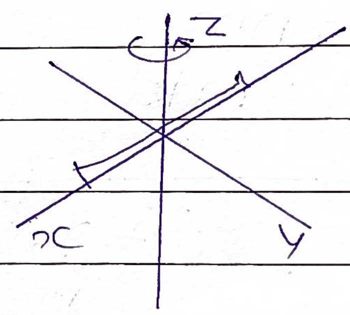
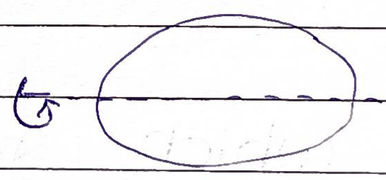
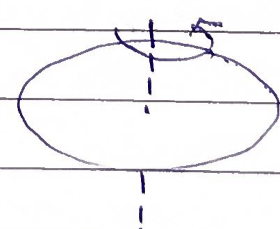
$M = nm$

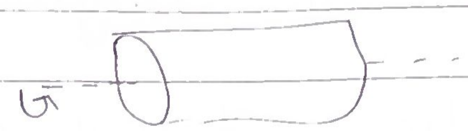
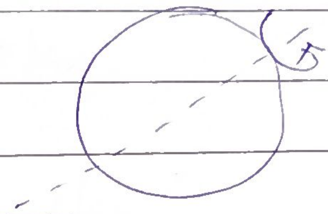
$I = mk^2$

$$\boxed{K^2 = \frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$

Factor on which radius of gyration of a body depend :-

Table :-
Moment of inertia of some regular shaped bodies -

S	Body	Axis	Figure	I
1	Thin circular ring, radius R	⊥r to plane at centre		MR^2
2	Thin circular ring, radius R	Diameter		$\frac{MR^2}{2}$
3	Thin rod length L	⊥r to rod at mid point		$\frac{Ml^2}{2}$
4	Circular disc radius R	Diameter		$\frac{MR^2}{4}$
5	Circular disc, Radius R	⊥r to disc at centre		$\frac{MR^2}{2}$
6	Hollow Cylinder, radius R	Axis of Cylinder		MR^2

7	Solid cylinder Radius R	Axis of cylinder		$\frac{MR^2}{2}$
8	Solid sphere radius R	Diameter		$\frac{2MR^2}{5}$

Relation b/w Torque & moment of Inertia

$$\tau = Fr$$

$$\tau = (ma)r$$
$$\tau = m(r\alpha)r$$
$$\tau = (mr^2)\alpha$$

$$\boxed{\tau = I\alpha}$$

Relation b/w angular momentum, moment of inertia and angular ~~mome~~ velocity.

$$L = Pr$$

$$L = (mv)r$$
$$L = m(r\omega)r$$

$$L = (mr^2)\omega$$

$$\boxed{L = I\omega}$$

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Conservation of Angulor momentum :-

Suppose the external torque acting on a rigid body due to its external force is zero then -

$$\tau = \frac{dL}{dt} = 0$$

$$\therefore dL = 0$$

$$L = \text{constant}$$

$$I\omega = \text{con.}$$

$$I_1\omega_1 = I_1\omega_1$$